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Remote controlling the dynamics of Bose–Einstein condensates through time-dependent atomic feeding and trap

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Abstract

In this paper, we generate the Lax pair of the nonconservative Gross–Pitaevskii (GP) equation with time-dependent linear and harmonic oscillator potentials and construct a multisoliton solution using gauge transformation. We show how an interplay between the dispersion coefficient, scattering length and atomic feeding can suitably be exploited to remote control the dynamics of solitons, thereby generating favorable profiles of Bose–Einstein condensates (BECs), notable among them being the matter wave similaritons.

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1. Introduction

The identification of Bose–Einstein condensates (BECs) [1–4] has given a new dimension to the field of condensed matter physics and atom optics, and has generated immense interest in understanding the dynamics of ultra-cold matter. In fact, the domain of BECs has virtually blossomed in the last decade or so, ever since the experimental realization of dark [5] and bright solitons [6], Faraday waves [7], gap solitons [8], etc.

It should be mentioned that the evolution of the macroscopic wavefunction of these condensates is described by the mean field Gross–Pitaevskii (GP) equation which is an inhomogeneous nonlinear Schrödinger (NLS) equation. The presence of the inhomogeneity in the GP equation which can be attributed either to the trapping potentials or to the nonuniformity of the media has always made the construction of analytic solutions a formidable exercise. Even though the Darboux transformation method [9] has recently been employed to generate soliton solutions [10], the construction of multisoliton solutions using this approach even today remains a challenging task besides being cumbersome. The gauge transformation approach

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[11] has come in handy at this juncture as it facilitates the construction of multisoliton solution from the associated linear eigenvalue problem starting from a trivial seed solution [12–14].

Recently, it was shown [10, 12] that when the binary interatomic interaction varies exponentially with time at a rate that equals the trapping potential strength, the matter wave densities can suitably be manipulated either to compress or broaden the condensates without causing either the explosion or the collapse of the condensates. This severe restriction would make the exact solutions of the GP equation less interesting from an experimental point of view. In addition, it would also be interesting to investigate the dynamics of the condensates under the impact of an external time-dependent (or constant) force to explore the possibility of controlling the evolution of condensates without the trapping potential. The present paper is aimed at softening the restriction arising from the dependence of interatomic interaction on the trapping potential strength as it tries to generate the exact analytic solutions of the variable coefficient GP equation in the presence of time-dependent linear and harmonic oscillator potentials by the addition of a nonconservative force originating by virtue of time-dependent atomic feeding. In particular, we attempt to generate localized excitations in BECs arising by virtue of the delicate balance existing between the strength of harmonic trap, scattering length, dispersion coefficient and atomic feeding. The fact that the above-mentioned parameters of the associated dynamical system are time dependent ensures that our results are tailor made for realistic experiments.

Considering the variable coefficient GP equation with time-dependent linear and harmonic oscillator potentials in the presence of time-dependent atomic feeding [15], we have

$$\mathrm{i}Q_t + \frac{D(t)}{2}Q_{xx} + \sigma R(t)|Q|^2 Q - 2\alpha(t)xQ - \frac{\Omega^2(t)}{2}x^2 Q + \frac{\mathrm{i}g(t)}{2}Q = 0 \tag{1}$$

where $D(t)$ and $R(t)$ represent dispersion coefficient and nonlinearity amplitude (interatomic interaction) respectively while $\Omega(t)$ describes the confining time-dependent harmonic trap and the coefficient of linear potential $\alpha(t)$ represents in general gravity of the dynamical system under consideration, $g(t)$ represents the time-dependent atomic feeding of the condensates from the thermal cloud. Investigation of this variable coefficient GP equation assumes tremendous significance from the perspective of nonlinear optics and condensed matter physics. From equation (1), we observe that when $D(t) = 1$, $\alpha(t) = 0$, $\Omega(t) =$ constant, $g(t) = 0$, it reduces to the GP equation which models the evolution of the condensates in the presence of harmonic potentials [10, 12]. When $D(t) = 1$, $\alpha(t) = 0$, $g(t) = 0$, it describes the dynamics of the condensates in a time-dependent trapping potential [14, 16, 17]. When $D(t) = 2$, $\alpha(t) = 1/2$, $R(t) =$ constant, $\Omega(t) = 0$ and $g(t) = 0$, it reduces to the GP equation with a linear potential [18]. When $\alpha(t) = 0$, $D(t) = R(t) = 1$, it reduces to the variable coefficient NLS equation admitting optical similaritons [19]. Thus, one understands that the investigation of equation (1) will open new avenues to perform realistic experiments in optical fibres and BECs.

2. Lax pair and multisoliton solution

Under the following transformation,

$$Q(x,t) = \sqrt{\frac{D(t)}{R(t)}}\exp\left[\frac{-\mathrm{i}x^2\Theta(t)}{2}\right]q(x,t) \tag{2}$$

equation (1) can be transformed into the following form,

$$\mathrm{i}q_t + \frac{D(t)}{2}q_{xx} + \sigma D(t)|q|^2 q - \mathrm{i}D(t)x\Theta q_x - \mathrm{i}D(t)\Theta q - 2\alpha(t)xq = 0, \tag{3}$$

where

$$\Theta(t) = \frac{D(t)R'(t) - R(t)D'(t)}{D(t)^2 R(t)} - \frac{g(t)}{D(t)} \tag{4}$$

$$\Omega^2(t)D(t) = \frac{D'(t)^2 - D(t)D''(t)}{D(t)^2} + \frac{R(t)R''(t) - 2R'(t)^2}{R(t)^2} + \frac{D'(t)R'(t)}{D(t)R(t)} - g'(t) - \frac{g(t)D'(t)}{D(t)} + \frac{2g(t)R'(t)}{R(t)} - g(t)^2. \tag{5}$$

Equation (5) suggests that a suitable interplay between dispersion coefficient $D(t)$, scattering length $R(t)$, atomic feeding $g(t)$ and time-dependent harmonic trap $\Omega(t)$ can generate various kinds of localized excitations in the condensates. The arbitrary nature of the scattering length $R(t)$, atomic feeding $g(t)$ and dispersion coefficient $D(t)$ as it is evident from equation (5) suggests that this model is tailor made for experiments.

Equation (3) admits the following linear eigenvalue problem,

$$\Phi_x = U\Phi, \qquad U = \begin{pmatrix} -\mathrm{i}\zeta(t) & \sqrt{\sigma}q(x,t) \\ -\sqrt{\sigma}q(x,t)^* & \mathrm{i}\zeta(t) \end{pmatrix} \tag{6}$$

$$\Phi_t = V\Phi$$

$$V = \mathrm{i}\begin{pmatrix} \frac{\sigma}{2}D(t)|q|^2 - \alpha x & \sqrt{\sigma}D(t)\left(\frac{1}{2}q_x - \mathrm{i}\Theta x q\right) \\ \sqrt{\sigma}D(t)\left(\frac{1}{2}q_x^* + \mathrm{i}\Theta x q^*\right) & -\frac{\sigma}{2}D(t)|q|^2 + \alpha x \end{pmatrix} - \mathrm{i}\zeta(t)D(t)\begin{pmatrix} \Theta x & \mathrm{i}\sqrt{\sigma}q \\ -\mathrm{i}\sqrt{\sigma}q^* & -\Theta x \end{pmatrix} - \mathrm{i}\zeta(t)^2 D(t)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{7}$$

where ζ is the complex nonisospectral parameter obeying the first-order linear differential equation of the form

$$\zeta_t = \alpha(t) + D(t)\Theta(t)\zeta \tag{8}$$

Thus, it is obvious that the compatability condition $U_t - V_x + [\mathrm{U,V}] = 0$ generates equation (3) (keeping in mind equation (8)). Considering a trivial seed solution $q^{(0)} = 0$ and employing the gauge transformation approach [11], we obtain the bright two soliton solution of equation (1) for $\sigma = 1$ as

$$Q^{(2)} = \sqrt{\frac{D(t)}{R(t)}}\left(\frac{A_1 + A_2 + A_3 + A_4}{B_1 + B_2 + B_3}\right)\mathrm{e}^{\left(\frac{-\mathrm{i}x^2}{2}\left(\frac{D(t)R'(t) - R(t)D'(t)}{D(t)^2 R(t)} - \frac{g(t)}{D(t)}\right)\right)} \tag{9}$$

where

$$A_1 = -2\beta_2(\zeta_2 - \mu_1)(\mu_2 - \mu_1)\,\mathrm{e}^{-\theta_1 - \mathrm{i}\psi_2},$$
$$A_2 = -2\beta_2(\mu_2 - \zeta_1)(\zeta_2 - \mu_1)\,\mathrm{e}^{\theta_1 - \mathrm{i}\psi_2},$$
$$A_3 = -2\beta_1(\mu_2 - \zeta_1)(\mu_2 - \mu_1)\,\mathrm{e}^{-\mathrm{i}\psi_1 - \theta_2},$$
$$A_4 = -4\mathrm{i}\beta_1\beta_2(\zeta_2 - \zeta_1)\,\mathrm{e}^{-\mathrm{i}\psi_1 + \theta_2},$$
$$B_1 = (\zeta_2 - \zeta_1)(\mu_2 - \mu_1)\cosh(\theta_1 + \theta_2),$$
$$B_2 = (\zeta_2 - \mu_1)(\mu_2 - \zeta_1)\cosh(\theta_1 - \theta_2),$$
$$B_3 = -4\beta_1\beta_2\cos(\psi_1 - \psi_2),$$

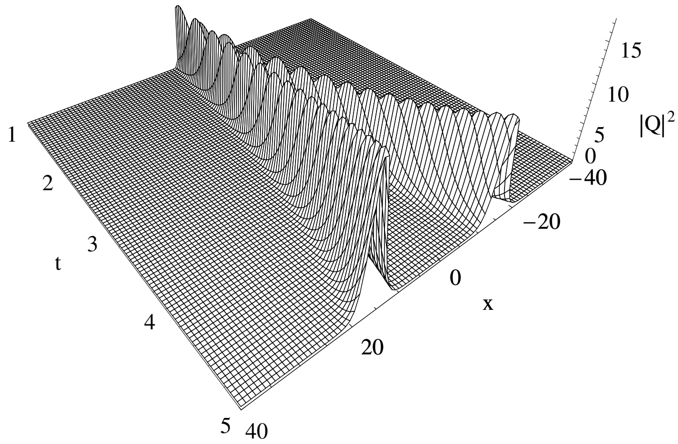

Figure 1. Effect of time-dependent trap on the dynamics of BECs with $g(t) = -0.15t$, $R(t) = \exp(-\int 0.25t\,dt)$, $D(t) = 1$, $\alpha(t) = 0$.

and

$$\zeta_i(t) = \alpha_i(t) + i\beta_i(t), \qquad \mu_i = \zeta_i, \qquad i = 1, 2$$

$$\alpha_i = \exp\int\left(-g(t) + \frac{R'(t)}{R(t)} - \frac{D'(t)}{D(t)}\right)dt \times\left[\alpha_0 + \int \mathrm{Exp}\left(\int g(t) - \frac{R'(t)}{R(t)} + \frac{D'(t)}{D(t)}\,dt\right)\alpha(t)\,dt\right]$$

$$\beta_i = \exp\int\left(-g(t) + \frac{R'(t)}{R(t)} - \frac{D'(t)}{D(t)}\right)\beta_0(t)\,dt$$

$$\theta_i = 2\beta_i x + 4\int D(t)\alpha_i(t)\beta_i(t)\,dt - 2\delta_i$$

$$\psi_i = 2\alpha_i x + 2\int D(t)[\alpha_i(t)^2 - \beta_i(t)^2]\,dt + 2\phi_i.$$

Looking at the nature of the soliton solution given by equation (9), one observes that an interplay between dispersion coefficient $D(t)$, scattering length $R(t)$ and atomic feeding $g(t)$ can be suitably exploited to remote control the dynamics of BECs.

3. Interaction of solitons and discussion

Case (i). When $D(t) = 1$, $\alpha(t) = 0$, the profile of the condensates is shown in figures 1 and 2. Figure 1 shows that there is an enhancement in the densities of solitons by virtue of the negative atomic feeding (gain) while the matter wave densities decrease due to the reversal of the sign of the atomic feeding as shown in figure 2. It can also be observed from figures 1 and 2 that even though the scattering length decreases exponentially with time, there is either compression (figure 1) or broadening (figure 2) of the solitons under the impact of atomic feeding (gain/loss) and this underscores the dominance of atomic feeding $g(t)$ over the binary interatomic interaction $R(t)$.

Case (ii). Figure 3 shows the effect of zero trapping potential (both linear and harmonic) on the evolution of the condensates. From this, one observes that one can enhance the

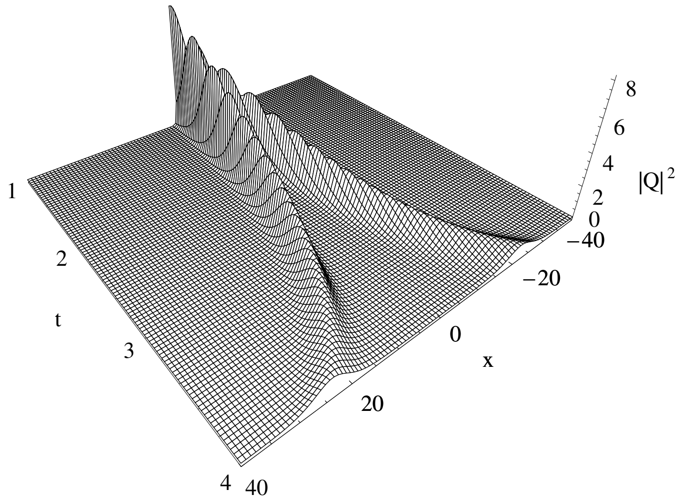

Figure 2. Effect of time-dependent trap on BECs with $g(t) = 0.01\mathrm{t}$, $R(t) = \exp(-\int 0.25\mathrm{t}\,\mathrm{dt})$, $D(t) = 1$, $\alpha(t) = 0$.

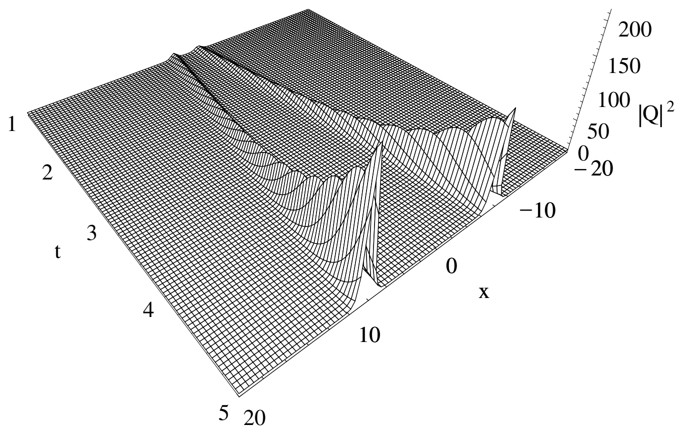

Figure 3. Matter wave similaritons under zero trapping potential with $g(t) = -0.25\mathrm{t}$, $R(t) = \exp(-\int 0.25\mathrm{t}\,\mathrm{dt})$, $D(t) = 1$, $\alpha(t) = 0$.

amplitude of solitons by suitably manoeuvring the atomic feeding thereby reinforcing the fact that stabilization of the condensates is indeed possible even without the trapping potentials by suitably feeding from the thermal cloud. The increase of amplitude of solitons as shown in figure 3 and the enhancement of width observed in figure 4 is reminiscent of optical similariton solutions [19], where the amplitude and width of the waves simply scale with time or propagation distance. We call these solutions 'matter wave similaritons'. Thus, we observe that it is possible to generate similaritons in BECs by suddenly switching off the time-dependent field (trap).

The moment the atomic feeding is switched off, the effect of binary interatomic interaction comes into play resulting in the dispersion of solitons. This is illustrated in figure 5 wherein the two solitons get dispersed and fizzle out in the absence of atomic feeding

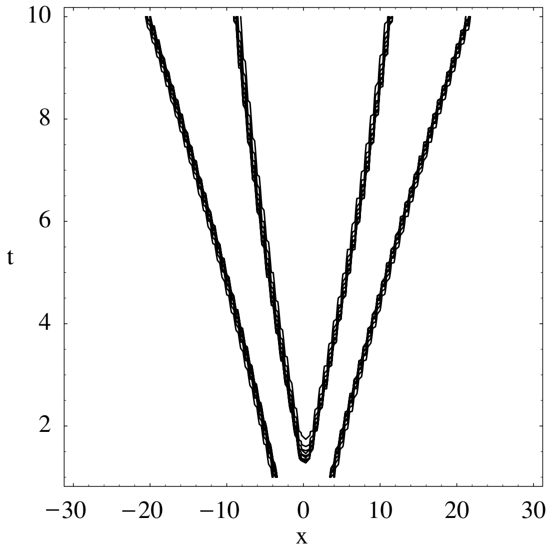

Figure 4. Contour plot of matter wave similaritons with $g(t) = -0.25\text{t}$, $R(t) = \exp(-\int 0.25\text{t}\,\text{dt})$, $D(t) = 1$, $\alpha(t) = 0$.

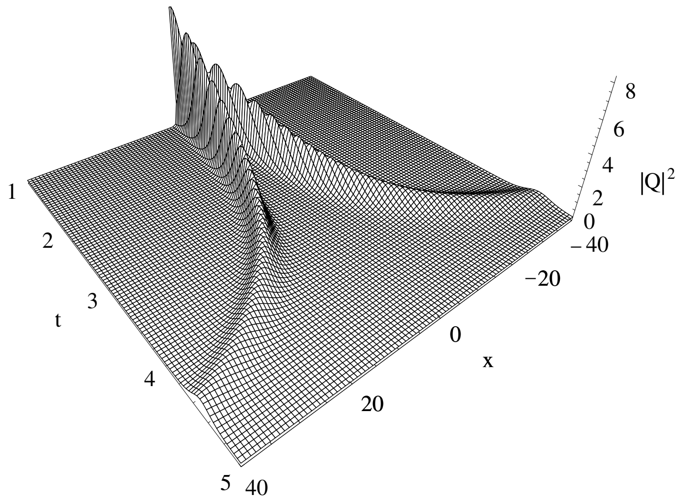

Figure 5. Effect of time-dependent harmonic trap and the absence of feeding on BECs with $g(t) = 0$, $R(t) = \exp(-\int 0.25\text{t}\,\text{dt})$, $D(t) = 1$, $\alpha(t) = 0$.

by virtue of the exponentially decreasing scattering length. It is worth noting at this juncture that for exponentially decreasing scattering lengths, the matter wave density of the condensates moving in a time-dependent trap [14] and harmonic potential [10, 12] decreases slowly.

Case (iii): Figures (6) and (7) bring out the effect of periodic and constant atomic feeding on Feshbach resonance management of condensates. Periodic feeding of BECs ensures that their density can be manipulated to reach a maximum value at regular intervals of time as shown in

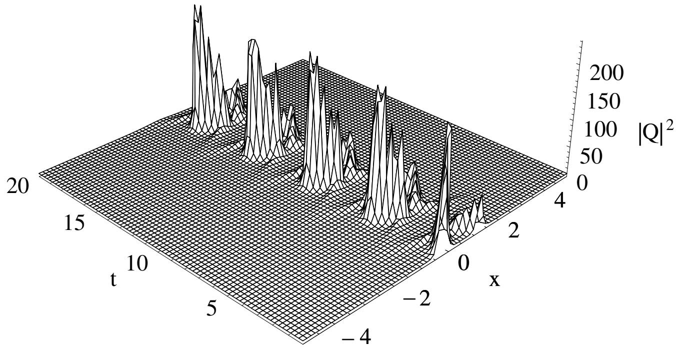

Figure 6. Feshbach resonance managed solitons with $g(t) = \sin(\sqrt{2}t)$, $R(t) = D(t) = \alpha(t) = 0.4\sin(\sqrt{2}t)$.

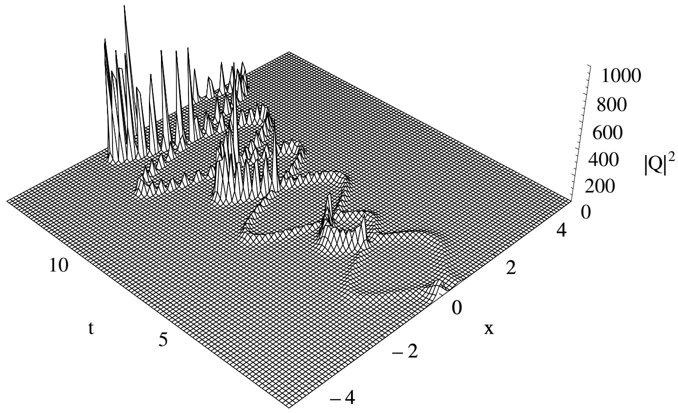

Figure 7. Feshbach resonance managed solitons with $g(t) = -0.1$, $R(t) = D(t) = \alpha(t) = 0.4\sin(\sqrt{2}t)$.

figure 6 while constant feeding of atoms shows that $|Q|^2$ continues to increase periodically as shown in figure 7.

Case (iv): Figure 8 shows the effect of gravity on the evolution of BECs through atomic feeding. From this, one observes that the trajectory of the two solitons gets curved and the density of the condensates increases.

From the above, it is evident that even though the variable coefficient GP equation (1) admits five parameters of different physical significance, the arbitrary time-dependent atomic feeding $g(t)$ prevails over the other parameters in controlling the dynamics of BECs. Hence, the quasi-one-dimensionality of the system which is related to the low densities of the condensates depends on how far one can feed the atoms to the condensates from the thermal cloud. Thus, by suitably exploiting the arbitrary nature of the atomic feeding $g(t)$ (consistent with equation (5)), one can ensure the quasi-one-dimensionality of the system under consideration.

In this paper, we have constructed the Lax pair of the nonconservative GP equation and have shown how the atomic feeding can suitably be manipulated to remote control the dynamics of Bose-Einstein condensates in a favourable manner. We have also brought out

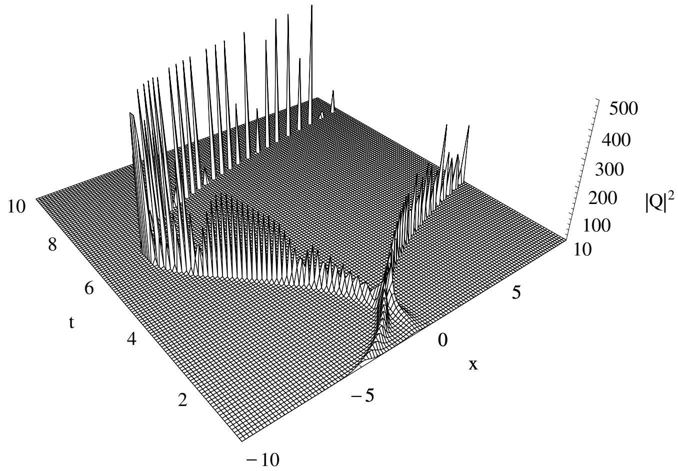

Figure 8. Effect of linear potential on BECs with $g(t) = -1/\text{t}$, $\alpha(\text{t}) = -0.2\text{t}$, $R(t) = 0.1\text{t}$, $D(t) = 0.2\text{t}$.

the dominance of atomic feeding $g(t)$ over the binary interatomic interaction $R(t)$ and the harmonic trap $\Omega(t)$. Our results reinforce the fact that the condensates can be stabilized even without the trap and this occurs no matter whether there is an increase or decrease of scattering length.

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